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Large Amplitude Free Vibration of Rectangular Plates Subjected to Aerodynamic Heating

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SUMMARY

Large amplitude free vibrations of rectangular plates with different boundaries and temperature distributions are analyzed in the light of Berger's analysis. A successive approximate method of Poincaré [10, 1] or a modified Galerkin technique is used to derive a Duffing type non-linear differential equation of which the solution is obtained by use of elliptic functions or by the use of method of successive approximation [7, 1].

The subject is treated in a simple and unified manner. Numerical results are obtained for different boundary conditions and are shown in graphical form.

1. Introduction

There is growing interest in the large deflection vibration problems of elastic plates particularly in the field of high speed aerodynamics. To the author's knowledge, only a few works are available in this direction. Chu and Hermann [15] used the perturbation method to solve the free large-deflection vibration of rectangular plate with hinged, immovable edges and with constant thickness. The exact solution for large-deflection vibration of the lenticular unsupported plates was discussed by Harris and Mansfield [12]. While Mansfield pointed out in his papers [13, 14] in 1962 and 1965 respectively about the possibility of exact solution of such a problem due to its lenticular section and unsupported edge, it should be noted that such exact solution is difficult for any supported plate with constant thickness. Sunakawa [1] used the Poincaré's method of successive approximation to solve the exact non-linear equations of heated rectangular plate with clamped or simply supported edges for both static and dynamic cases. But in all these studies, the fundamental equations of equilibrium or motion are non-linear and coupled in character and as such they are difficult to solve.

An approximate method for investigating the large deflections of initially flat isotropic plates has been proposed by Berger [4]. Though this method which consists of neglecting the so-called second strain invariant of the middle plane in the expression for strain energy, lacks sufficient technical interpretation for its justification, this analysis yields results agreeing with known studies for all practical purposes [4]. Since then a number of static and dynamic problems have been solved with remarkable ease and satisfactory approximation by using this method [2, 3, 5].

The purpose of this investigation is to use Berger's technique of neglecting the second strain invariant of the middle surface, for the study of large amplitude free vibrations of heated rectangular plates with different boundary conditions.

In this paper, the author has derived the analogue of Berger's approximation equations for the large amplitude free vibration of a heated rectangular plate. These approximate, decoupled equations are solved by employing either successive approximation method or Galerkin's technique. Numerical results are obtained on the digital computer IBM 1620, for different boundary conditions and temperature distributions. Comparisons with the known results of [1] asserts the usefulness of Berger's approach to yield results sufficient for all practical purposes.

Notation

$2a$, $2b$, d	length, width and thickness of the rectangular plate, respectively
t	time
u, v, w	displacement components in the middle plane in the x-, y- and z-directions respectively
$z(\theta, t)$	displacement at the centre of the plate
z_1, z_2	absolute values of the maximum and minimum non-dimensional amplitude respectively
D	flexural modulus of rigidity $D=\dfrac{Eh^3}{12(1-\nu^2)}$
E, G	Young's modulus of elasticity and shear modulus, respectively
$\bar{E}$	total energy of the vibrating system
$K(K_1)$	complete elliptic integral of the first kind
T, T^*	linear and non-linear periods respectively
α, ρ	coefficient of linear thermal expansion and density of the plate material
k^2	constant of integration
τ, ξ, ζ	non-dimensional time
θ	temperature change from the initial state
$\bar{\theta}$, $\tilde{\theta}$	mean temperature and temperature moment
λ	aspect ratio of the rectangular plate
ν	Poisson's ratio
χ	Airy's stress function
ω, ω^*	linear and non-linear circular frequency respectively
∇^2, ∇^4	harmonic and biharmonic operators respectively

Note: i) Subscripts "x" and "y" denote the partial differentiation with respect to x and y respectively.
ii) Subscripts "s", "c" and "m" specify the quantity for the cases of the simply supported on all edges, clamped on all edges and simply supported on two opposite edges and clamped on other edges respectively.

2. Fundamental Equations

The fundamental equations of motion of a heated rectangular plate due to Berger's analysis are

$$D\nabla^4 w - k^2\nabla^2 w + \frac{E\alpha d^2}{1-\nu}\nabla^2\tilde{\theta} = -\rho d\frac{\partial^2 w}{\partial t^2} \tag{1}$$

$$\frac{12D}{d^2}e_1 - \frac{E\alpha d}{1-\nu}\bar{\theta} = k^2 \tag{2}$$

where

$$e_1 = u_x + v_y + \tfrac{1}{2}(w_x^2 + w_y^2) \tag{3}$$

$$\bar{\theta} = \frac{1}{d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\theta(x, y, z)\,dz \tag{4}$$

$$\tilde{\theta} = \frac{1}{d^2}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}z\theta(x, y, z)\,dz\,. \tag{5}$$

In the derivation of the above equations, the effect of the internal friction, aerodynamic force and inertia effects in the plate plane are neglected.

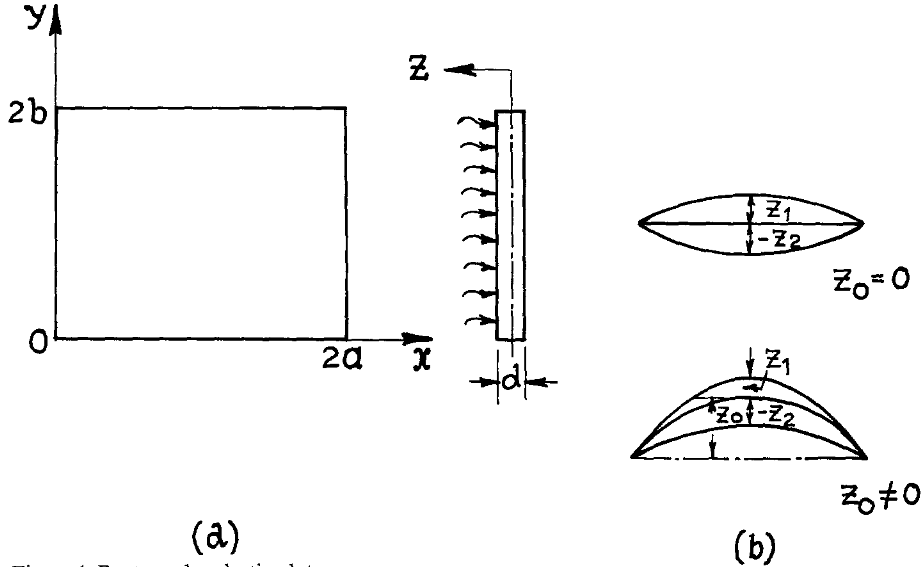

Figure 1. Rectangular elastic plate.

Boundary conditions for different cases are given as follows:

(i) Simply supported on all edges,

$$\left.\begin{aligned} &w=0 \\ &D(w_{xx}+\nu w_{yy}) + \frac{Ed^2\alpha\tilde{\theta}}{1-\nu} = 0 \end{aligned}\right\} \text{at } x=0,\, 2a \tag{6a}$$

$$\left.\begin{aligned} &w=0 \\ &D(w_{yy}+\nu w_{xx}) + \frac{Ed^2\alpha\tilde{\theta}}{1-\nu} = 0 \end{aligned}\right\} \text{at } y=0, 2b\,. \tag{6b}$$

(ii) Clamped on all edges

$$\begin{aligned} &w = w_x = 0 \quad \text{at} \quad x = 0,\, 2a \\ &w = w_y = 0 \quad \text{at} \quad y = 0,\, 2b \end{aligned} \tag{7}$$

(iii) Simply supported on a pair of opposite edges and remaining edges clamped

$$\left.\begin{aligned} &w=0 \\ &D(w_{xx}+\nu w_{yy}) + \frac{Ed^2\alpha}{1-\nu}\tilde{\theta} = 0 \end{aligned}\right\} \text{at } x=0,\, 2a \tag{8a}$$

$$w = w_y = 0 \quad \text{at} \quad y = 0,\, 2b\,. \tag{8b}$$

The temperature distribution over the plate is assumed to be symmetrical with respect to the centre of plate and is given as

$$\bar{\theta} = \sum_i^{\infty}\sum_j^{\infty} \bar{\theta}_{ij} \cos\frac{i\pi x}{2a} \cos\frac{j\pi y}{2b}, \quad (i, j = 0, 2, 4, \ldots \text{ even}) \tag{9}$$

$$\tilde{\theta} = \sum_p^{\infty}\sum_q^{\infty} \tilde{\theta}_{pq} \sin\frac{p\pi x}{2a} \sin\frac{q\pi y}{1b}, \; (p, q = 1, 3, 5, \ldots \text{ odd})\,. \tag{10}$$

It is assumed in equation (10) that there exists no temperature gradient through the thickness at the edges. Since it seems to be natural to expect that there exists no remarkable difference between the wave form of the present non-linear vibration and that of the small vibration [11], the lowest mode of vibration is assumed to be the same as the deflection form due to the temperature change only, and the normal displacement of the plate at any time is assumed as the sum of the displacement due to the temperature change and the amplitude of vibration, as

$$w(x, y; \theta, t) = z(\theta, t)\, w_i(x, y) \tag{11}$$

where

$$\frac{z(\theta, t)}{d} = z_0(\theta) + [\bar{z}(t)]_{\theta=\text{const.}} \tag{12}$$

$$w_s(x, y) = \sin\frac{\pi x}{2a}\sin\frac{\pi y}{2b} \tag{13}$$

$$w_c(x, y) = \frac{1}{4}\left(1-\cos\frac{\pi x}{a}\right)\left(1-\cos\frac{\pi y}{b}\right) \tag{14}$$

$$w_m(x, y) = \frac{1}{2}\sin\frac{\pi x}{2a}\left(1-\cos\frac{\pi y}{b}\right). \tag{15}$$

The expressions (13), (14) and (15) for w_i satisfy the boundary conditions (6), (7) and (8) respectively.

Using the expressions for $\bar{\theta}$ and w in equation (2) and integrating throughout the plate, we obtain the value of k^2 supposing that there are no in-plane displacements at the edges of the plate and these are as follows:

$$k_s^2 = \tfrac{3}{2}D\left(\frac{z}{d}\right)^2\left\{\left(\frac{\pi}{2a}\right)^2 + \left(\frac{\pi}{2b}\right)^2\right\} - \frac{E\alpha d}{1-\nu}\bar{\theta}_{00}$$

$$k_c^2 = \tfrac{9}{8}D\left(\frac{z}{d}\right)^2\left\{\left(\frac{\pi}{2a}\right)^2 + \left(\frac{\pi}{2b}\right)^2\right\} - \frac{E\alpha d}{1-\nu}\bar{\theta}_{00}$$

$$k_m^2 = \tfrac{3}{8}D\left(\frac{z}{d}\right)^2\left\{3\left(\frac{\pi}{2a}\right)^2 + 4\left(\frac{\pi}{2b}\right)^2\right\} - \frac{E\alpha d}{1-\nu}\bar{\theta}_{00}\,.$$

For the case, when the plate is simply supported on all edges, the term $\tilde{\theta}_e$ (temperature moment at the edges) is added to the right-hand side of equation (10) for the general expression of the temperature moment. The equation of motion (1) is solved by a successive approximation method due to Poincaré [10] satisfying the boundary conditions (6), which finally gives the non-linear differential equation (16). However, the presence of $\tilde{\theta}_e$ does not introduce any new phenomena in the present dynamic problem, but only results in complicating the calculations; so the equation (16) is obtained from equation (1) for the other two cases of boundary conditions, that is clamped on all edges and two opposite edges simply supported and remaining edges clamped omitting the term $\tilde{\theta}_e$ and using the modified Galerkin method only

$$\frac{d^2\bar{z}}{d\tau^2} + f_3^1(z_0+\bar{z})^3 + f_1^1(z_0+\bar{z}) = g\,. \tag{16}$$

Eliminating the part in equation (16) corresponding to the deflection state due to the temperature change only, we have the following equation corresponding to the vibration state:

$$\frac{d^2\bar{z}}{d\tau^2} + (f_1^1 + 3f_3^1 z_0^2)\bar{z} + 3f_3^1 z_0\bar{z}^2 + f_3^1\bar{z}^3 = 0\,.$$

The equation corresponding to the deflection state due to the temperature change is

$$f_1^1 z_0 + f_3^1 z_0^3 = g \tag{18}$$

where

$$\tau = \frac{1}{(2b)^2}\sqrt{\frac{D}{\rho d}}\, t \tag{19}$$

$$f^1_{1,s} = \pi^4\left(1+\frac{1}{\lambda^2}\right)^2\left\{1-\frac{12(1+\nu)}{\pi^2}\cdot\frac{\lambda^2}{1+\lambda^2}\left(\frac{2b}{d}\right)^2\alpha\bar{\theta}_{00}\right\}$$

$$f^1_{1,c} = \frac{16\pi^4}{9}\left(\frac{3+2\lambda^2+3\lambda^4}{\lambda^4}\right) - 16(1+\nu)\alpha\pi^2\left(\frac{2b}{d}\right)^2\left(1+\frac{1}{\lambda^2}\right)\bar{\theta}_{00} \tag{21}$$

$$f^1_{1,m} = \frac{16\pi^4}{3}\left(1+\frac{1}{2\lambda^2}+\frac{3}{16\lambda^4}\right) - 16(1+\nu)\alpha\pi^2\left(\frac{2b}{d}\right)^2\left(1+\frac{3}{4\lambda^2}\right)\bar{\theta}_{00} \tag{22}$$

$$f^1_{3,s} = \frac{3\pi^4}{2}\left(1+\frac{1}{\lambda^2}\right)^2 \tag{23}$$

$$f^1_{3,c} = \frac{3\pi^4}{2}\left(1+\frac{1}{\lambda^2}\right)^2 \tag{24}$$

$$f^1_{3,m} = \frac{\pi^4}{8}\left(\frac{3}{\lambda^2}+4\right)^2 \tag{25}$$

$$g_s = 12(1+\nu)\pi^2\left(1+\frac{1}{\lambda^2}\right)^2\alpha\left[\sum_p^\infty\sum_q^\infty(-1)^{\frac{1}{2}(p+q)-1}\frac{\lambda^2\tilde{\theta}_{pq}}{p^2+\lambda^2q^2}\right.$$
$$\left. +\tilde{\theta}_e\sum_{m:\text{odd}}^\infty\frac{(-1)^{\frac{1}{2}(m-1)}}{m^2}\lambda\left\{\frac{\tanh\mu_m}{\cosh\mu_m}+\frac{\tanh\lambda^2\mu_m}{\cosh\lambda^2\mu_m}\right\}\right]\left(\frac{2b}{d}\right)^2 \tag{16a}$$

$$\mu_m = \frac{m\pi}{2\lambda} \tag{26b}$$

$$g_c = \frac{4096}{3}(1+\nu)\left(\frac{2b}{d}\right)^2\alpha\sum_p^\infty\sum_q^\infty\tilde{\theta}_{pq}\frac{p^2+\lambda^2q^2}{\lambda^2pq(p^2-4)(q^2-4)} \tag{27}$$

$$g_m = -128(1+\nu)\alpha\pi\left(\frac{2b}{d}\right)^2\sum_{q:\text{odd}}^\infty\tilde{\theta}_{1q}\frac{1+\lambda^2q^2}{\lambda^2q(q^2-4)}. \tag{28}$$

Equation (17) is the equation of vibration. Equation (18) gives $z_0(\theta)$ which is the static solution of the problem. The solutions of equation (18) have already been given by the present author in [9] and the analytical solution of equation (17) is given in the following section.

3. Analytical Solution

Using the relation

$$\xi = \sqrt{f^1_1+3f^1_3z_0^2}\,\tau \tag{29}$$

equation (17) is reduced to

$$\frac{d^2\bar{z}}{d\xi^2}+\bar{z}+f_2\bar{z}^2+f_3\bar{z}^3=0 \tag{30}$$

where

$$f_2 = \frac{3f^1_3z_0}{f^1_1+3f^1_3z_0^2}$$
$$f_3 = \frac{f^1_3}{f^1_1+3f^1_3z_0^2} \tag{31}$$

The solution of equation (30) may be obtained in terms of the incomplete elliptic integral [7, 12], but such solution is complicated and not suitable for practical applications and therefore, a method of successive approximation [7] is used to obtain the required solution.

Now, by using the transformation

$$\zeta = \sqrt{1+\beta}\,\xi \tag{32}$$

equation (30) is transformed into:

$$(1+\beta)\frac{d^2\bar{z}}{d\zeta^2} + \bar{z} = -f_2\bar{z}^2 - f_3\bar{z}^3 \tag{33}$$

Let z_1 and $-z_2$ be the maximum and minimum amplitude respectively of the displacement $\bar{z}$; then, β and $\bar{z}$ are expanded in the power series of z_2 as

$$\beta = -\beta_1 z_2 + \beta_2 z_2^2 - \beta_3 z_2^3 + \dots \tag{34}$$

$$\bar{z} = -\eta_1(\zeta) z_2 + \eta_2(\zeta) z_2^2 - \eta_3(\zeta) z_2^3 + \dots \tag{35}$$

Substituting equations (34) and (35) in equation (33) and equating the coefficient of z_2 and its higher powers to zero, we obtain a set of differential equations in $\eta_1, \eta_2, \eta_3, \dots$ with coefficients consisting of $\beta_1, \beta_2, \beta_3, \dots$. Solving these equations by the successive approximation method [1, 7], under the initial conditions $\eta_1(0)=1$, $\eta_2(0)=\eta_3(0)=\dots=0$ and $\dot{\eta}_1(0)=\dot{\eta}_2(0)=\dot{\eta}_3(0)=\dots=0$, we can get the values of $\beta_1, \beta_2, \beta_3, \dots$ and $\eta_1, \eta_2, \eta_3, \dots$. Thus, the solution of equation (33) in the final form becomes

$$\begin{aligned}
\bar{z} = {} & \{-\tfrac{1}{2}f_2 z_2^2 + \tfrac{1}{3}f_2^2 z_2^3 - (\tfrac{25}{48}f_2^3 - \tfrac{21}{32}f_2 f_3) z_2^4 + (\tfrac{25}{36}f_2^4 - \tfrac{29}{24}f_2^2 f_3) z_2^5 - \dots\} + \\
& \{-z_2 + \tfrac{1}{3}f_2 z_2^2 - (\tfrac{29}{144}f_2^2 - \tfrac{1}{32}f_3) z_2^3 + \tfrac{119}{432}f_2^3 - \tfrac{35}{96}f_2 f_3) z_2^4 + \\
& \qquad - (\tfrac{6971}{20736}f_2^4 - \tfrac{1475}{2304}f_2^2 f_3 + \tfrac{23}{1024}f_3^2) z_2^5 + \dots\} \cos\zeta + \\
& \{\tfrac{1}{6}f_2 z_2^2 - \tfrac{1}{9}f_2^2 z_2^3 + (\tfrac{2}{9}f_2^3 - \tfrac{1}{3}f_2 f_3) z_2^4 - (\tfrac{8}{27}f_2^4 - \tfrac{5}{9}f_2^2 f_3) z_2^5 + \dots\} \cos 2\zeta + \\
& \{-(\tfrac{1}{48}f_2^2 + \tfrac{1}{32}f_3) z_2^3 + (\tfrac{1}{48}f_2^3 + \tfrac{1}{32}f_2 f_3) z_2^4 - (\tfrac{31}{576}f_2^4 - \tfrac{11}{384}f_2^2 f_3 - \tfrac{3}{128}f_3^2) z_2^5 + \dots\} \\
& \qquad \cos 3\zeta + \\
& \{(\tfrac{1}{432}f_2^3 + \tfrac{1}{96}f_2 f_3) z_2^4 - (\tfrac{1}{324}f_2^4 + \tfrac{1}{72}f_2^2 f_3) z_2^5 + \dots\} \cos 4\zeta + \\
& \{-(\tfrac{5}{20736}f_2^4 + \tfrac{5}{2304}f_2^2 f_3 + \tfrac{1}{1024}f_3^2) z_2^5 + \dots\} \cos 5\zeta + \dots
\end{aligned} \tag{36}$$

Equation (36) is the solution of equation (33) when the amplitude of vibration is expressed as the function of z_2 and ζ. Here, for the case of infinitesimal value of z_2, the constant term and the higher harmonic terms in equation (36) can be neglected except only the fundamental harmonic term which is the solution of the linear theory. Equation (36) is the periodic function with respect to ζ with the period, 2π.

The period of the motion is given by

$$\begin{aligned}
T^*(t) = {} & \frac{2\pi(2b)^2}{\sqrt{f_1^1 + 3f_3^1 z_0^2}}\sqrt{\frac{\rho d}{D}}\,\{1 + (\tfrac{5}{12}f_2^2 - \tfrac{3}{8}f_3) z_2^2 - (\tfrac{5}{18}f_2^3 - \tfrac{1}{4}f_2 f_3) z_2^3 + \\
& + (\tfrac{385}{576}f_2^4 - \tfrac{275}{192}f_2^2 f_3 + \tfrac{57}{256}f_3^2) z_2^4 - \dots\}\,.
\end{aligned} \tag{37}$$

Equation (36) and (37) give the amplitude and period of the nonlinear vibration of the above mentioned plate subjected to the change in temperature. The period is a function of the amplitude, which is the characteristic of the non-linear vibration, and changes with the thermal stress and deflection due to the temperature change. Next, the relation between the maximum and minimum values of amplitude, z_1 and $-z_2$, is given below:

Applying the so-called energy integral to equation (30), the following equation is obtained

$$\left(\frac{d\bar{z}}{d\xi}\right)^2 + \bar{z}^2 + \tfrac{2}{3}f_2\bar{z}^3 + \tfrac{1}{2}f_3\bar{z}^4 = 2\bar{E}\,. \tag{38}$$

Here, $\bar{E}$ is the total energy of the vibrating system.

For extremum $d\bar{z}/d\xi = 0$ at $\bar{z} = z_1, -z_2$ and this condition reduces equation (38) to

$$z_1^2(1+\tfrac{2}{3}f_2z_1+\tfrac{1}{2}f_3z_1^2) = z_2^2(1-\tfrac{2}{3}f_2z_2+\tfrac{1}{2}f_3z_2^2) = 2\bar{E} \tag{39}$$

z_1 and z_2 can be determined independently whenever $\bar{E}$ is given in accordance with the initial conditions.

For the pre-buckling state, equation (30) is reduced to

$$\frac{d^2\bar{z}}{d\tau^2} + f_1^1\bar{z} + f_3^1\bar{z}^3 = 0 \tag{40}$$

for, in the pre-buckling state, $z_0 = 0$ gives $f_2 = 0$. Therefore, through the energy integral, equation (40) is reduced to

$$\tau = \pm\int_0^{\bar{z}} \frac{d\bar{z}}{\sqrt{2\bar{E}f_1^1 - f_1^1\bar{z}^2 - \tfrac{1}{2}f_3^1\bar{z}^4}}$$

$$= \pm\frac{1}{\sqrt{f_1^1+f_3^1z_2^2}}\int_0^{\phi}\frac{d\phi}{\sqrt{1-K_1^2\cos^2\phi}}$$

where

$$\frac{\bar{z}}{z_2} = \sin\phi, \quad K_1^2 = \frac{f_3^1z_2^2}{2(f_1^1+f_3^1z_2^2)}$$

The sign of the above integral is taken to be positive or negative according as $\bar{z}$ increases or decreases with the increase of τ. The period of the non-linear vibration, T^* is given by

$$T^*(t) = \frac{4(2b)^2}{\sqrt{f_1^1+f_3^1z_2^2}}\sqrt{\frac{\rho d}{D}}\,K(K_1) \tag{41}$$

where $K(K_1)$ is the complete elliptic integral of the first kind.

4. Numerical Example

Let us assume the temperature distribution on the elastic plates as

$$\left.\begin{aligned} &\theta = \bar{\theta} = \Theta \sin\frac{\pi x}{2a}\sin\frac{\pi y}{2b} \\ &\tilde{\theta} = 0 \quad \text{that is} \quad g = 0 \end{aligned}\right\} \tag{42}$$

The plate buckles at the critical temperature θ_{cr}, and so its vibration behaviour is studied separately for each of the cases before and after the thermal buckling.

Equations (20), (21), (22) with the help of equation (42) give

$$f_{1,s}^1 = \pi^4\left(1+\frac{1}{\lambda^2}\right)^2\left\{1 - \frac{64}{\pi^4}\frac{\lambda^2}{1+\lambda^2}\left(\frac{2b}{d}\right)^2\alpha\Theta\right\} \tag{43a}$$

$$f_{1,c}^1 = \frac{16\pi^4}{9}\frac{3+2\lambda^2+3\lambda^4}{\lambda^4} - \frac{256}{3}\left(1+\frac{1}{\lambda^2}\right)\left(\frac{2b}{d}\right)^2\alpha\Theta \tag{43b}$$

$$f_{1.m}^1 = \frac{16\pi^4}{3}\left(1+\frac{1}{2\lambda^2}+\frac{3}{16\lambda^4}\right) - \frac{256}{3}\left(1+\frac{3}{4\lambda^2}\right)\left(\frac{2b}{\mathrm{d}}\right)^2\alpha\Theta \tag{43c}$$

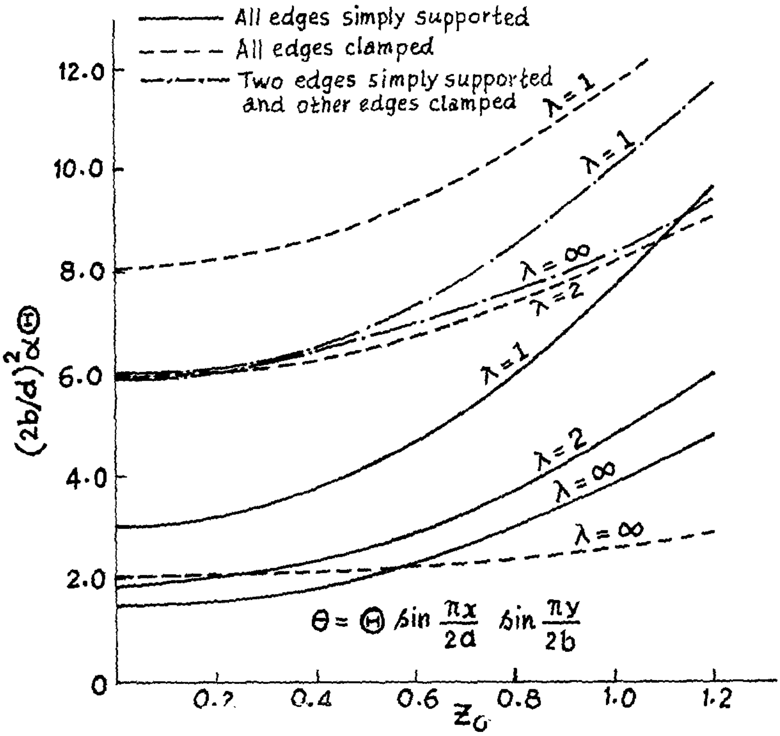

Figure 2. Relation between temperature rise and deflection at the centre of plate.

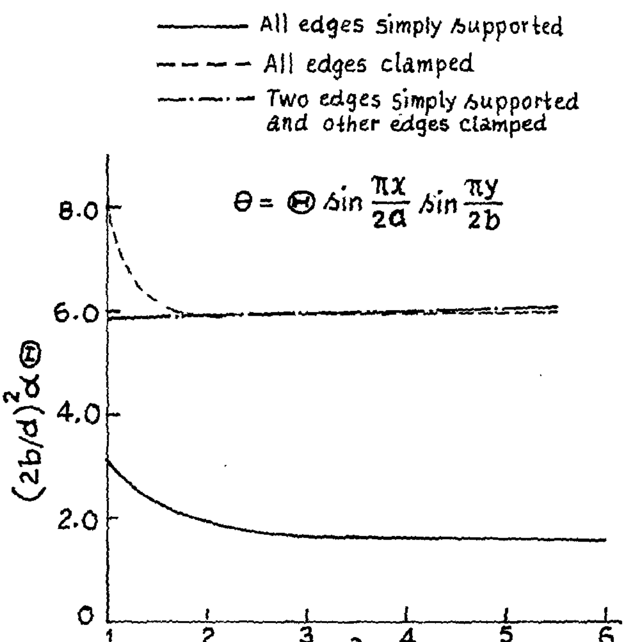

Figure 3. Variation of thermal buckling coefficient with aspect ratio.

The critical temperature Θ_{cr} and the deflection at the centre of the plate after buckling are obtained with the help of equations (18) and (42) and shown in Fig. 2 for different cases. The relations between the temperature and the aspect ratio λ are also presented in Fig. 3. The behaviour of the plates before and after buckling is discussed below.

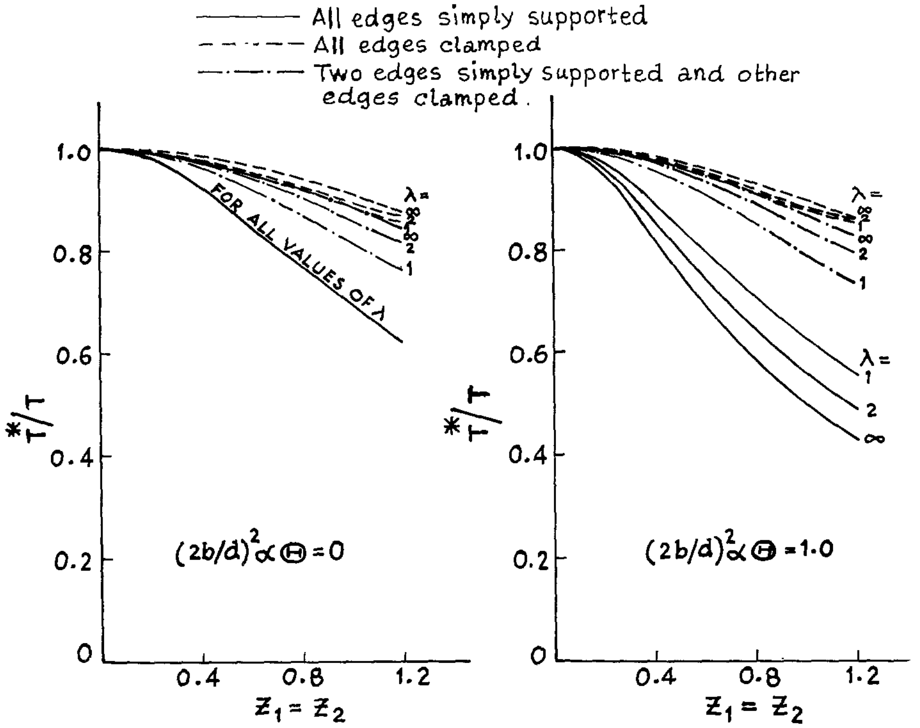

Figure 4. Influence of large amplitude on period of vibration of rectangular plates.

4.1. *When* $\Theta < \Theta_{cr}(z_0 = 0)$

The variation of T^*/T with the amplitude and temperature is given by equation (37) or (41) and show in Fig. 4 for different edge conditions. It is noted that the results agree with those obtained by Thein Wah in [2]. The variation of the circular frequency with the temperature is shown in Figs. 5, 6 and 7 for different boundary conditions with the amplitude as parameter.

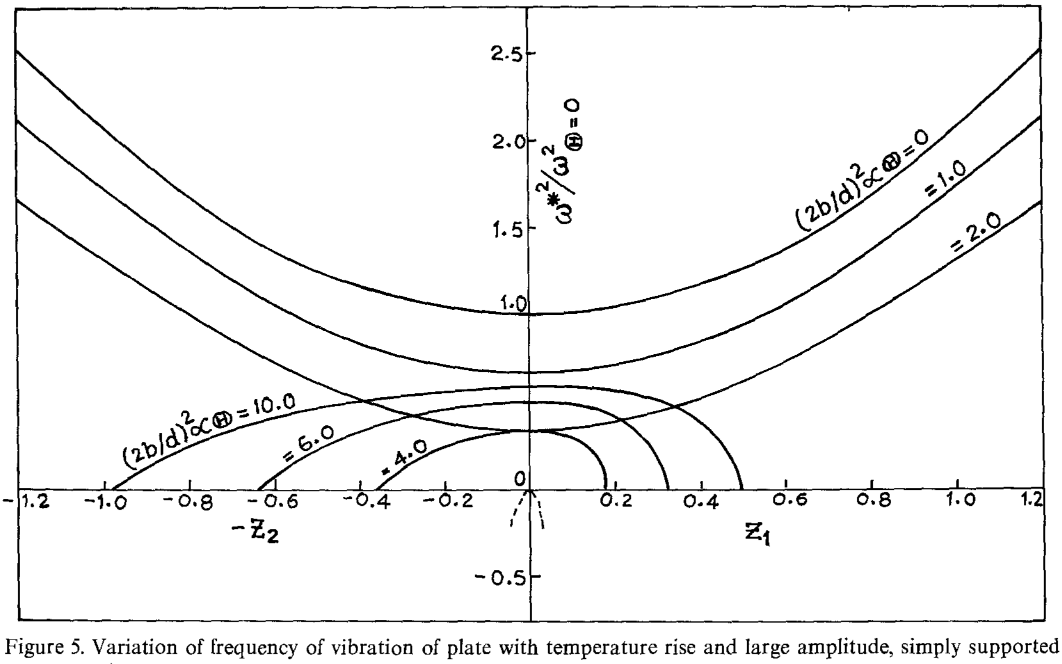

Figure 5. Variation of frequency of vibration of plate with temperature rise and large amplitude, simply supported edges ($\lambda = 1$).

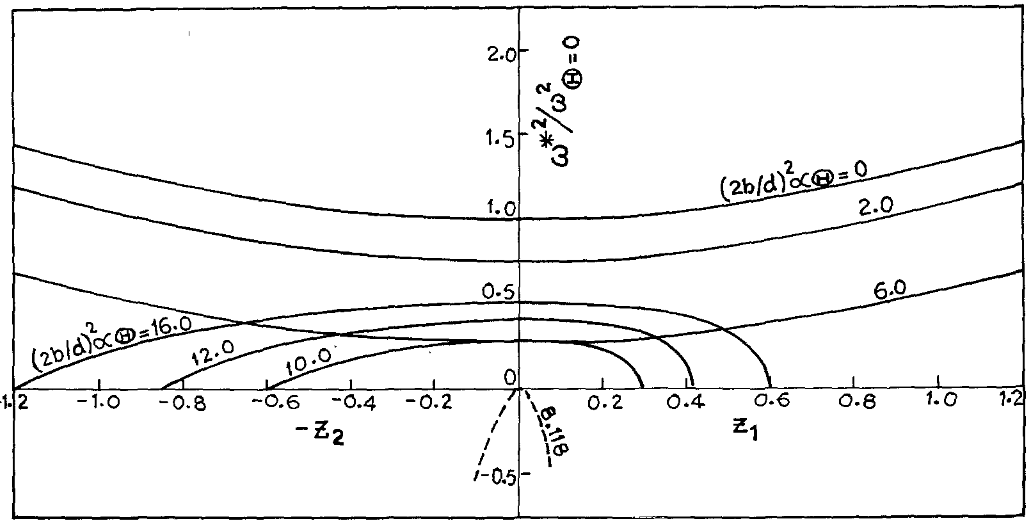

Figure 6. Variation of frequencies of vibration of plate with temperature rise and large amplitude, clamped on all edges ($\lambda = 1$).

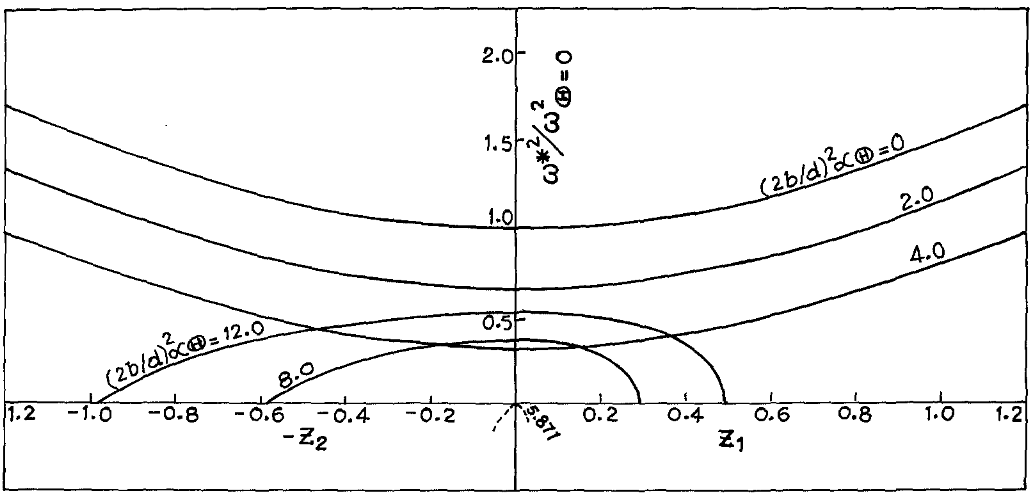

Figure 7. Variation of frequencies of vibration of plate with temperature rise and large amplitude, simply supported on two opposite edges and other edges clamped ($\lambda = 1$).

It is observed that the circular frequency for pre-buckling state decreases with the increase of temperature and increases with the increase of amplitude. This is in agreement with Sunakawa in [1].

4.2. *When* $\Theta > \Theta_{cr}(z_0 \neq 0)$

Using the values of z_0 from equations (18) and (42) in equation (37), the variation of the circular frequency with the temperature for various values of amplitude is obtained for $\lambda = 1$ and the behaviour is shown in Figs. 5, 6 and 7 for different edge conditions. Here, the circular frequency increases with the increase in temperature and decreases with the increase in amplitude when the temperature is constant, as observed by Sunakawa in [1]. This characteristic is opposed to that of the pre-buckling state.

During the post-buckling state, Figs. 5, 6 and 7 show that the non-linear free vibration ceases to occur when $z_2 > z_0$ or at that maximum value of z_2 corresponding to which the curve for $\Theta_1 > \Theta_{cr}$ intersects the abscissa at certain temperature. At this maximum absolute value of minimum amplitude, the plate suddenly jumps from one position of equilibrium $\bar{z}=0$ to the new position of equilibrium $\bar{z} = -2z_0$ (obtained from equation (38)), that is, the snap-through phenomenon takes place. After the occurrence of such a phenomenon, the plate will start to vibrate about the new position of equilibrium.

The above results are shown when the direction of the deflection due to buckling is towards the upper side. Due to symmetry, a similar nature of vibration can also take place when the direction of the deflection due to buckling is towards the lower side.

5. Conclusions

Berger's analysis has been applied in deriving the simplified, decoupled equation of motion for large amplitude free vibration of heated rectangular plates. Poincaré's successive approximation method in [10] or Galerkin's technique is used to derive a non-linear differential equation of Duffing type which is solved by the use of either successive approximation or elliptic integrals [7].

Numerical results are obtained for the vibration of *a rectangular flat plate* subjected to heating with different boundary conditions such as (a) all edges simply supported, (b) all edges clamped and (c) two opposite edges simply supported and other edges clamped. The results are in remarkable agreement with the already known results in [1].

An analysis, such as treated here, will be of considerable importance for supersonic airplanes and missiles, especially in studying the non-linear transient phenomena of vibration of structural components subjected to the thermal shock. It is also evident that Berger's approach presented in this paper yields results entirely adequate for many engineering applications.

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REFERENCES

[1] M. Sunakawa, Influences of temperature change and large amplitude on free flexural vibration of rectangular elastic plates, *ISAS*, Tokyo University, report no. 402, 31, 3, (June 1966).
[2] Wah Thein, Large amplitude flexural vibration of rectangular plates, *Int. J. Mech. Sci.*, 5, p. 425 (1963).
[3] N. Gajendar, Large amplitude vibration of plates on elastic foundations, *Int. J. Nonlinear Mechanics*, 2, p.163, (1967).
[4] H. M. Berger, A new approach to the analysis of large deflections of plates, *J. Appl. Mech.*, 22, p. 456, (1955).
[5] W. A. Nash, and J. R. Modeer, Certain approximate analyses of non-linear behaviour of plates and shellow shells, *Proc. Symp. on the theory of thin elastic shells*, Delft, The Netherlands, Vol. XIV, August, 1959.
[6] S. Timoshenko, and S. Woinowsky-Krieger, *Theory of plates and shells*, 2nd ed., McGraw-Hill, New York, 1959.
[7] S. Timoshenko, *Vibration problems in engineering*, D. Von Nostrand Company, Inc., 1961.
[8] W. Nowacki, *Thermoelasticity*, Addison–Wesley Publishing Company, Inc. London.
[9] M. C. Pal, *Large deflections of heated rectangular plates* (to appear).
[10] H. Poincaré, *Les Méthodes de la Méchanique Céleste*, Paris, Vol. 1, p. 32 (1892).
[11] Y. Yoshimura, and M. Uemura, *The effect of amplitude on the lateral vibration of thin beams*, Inst. Sci. Tech. Univ. of Tokyo, Report, 2 (1948) 57.
[12] G. Z. Harris and E. H. Mensfield, On the large-deflection vibrations of elastic plates, *Philosophical Transactions of the Royal Society of London*. A 261, 289–343 (1967).
[13] E. H. Mansfield, Bending, buckling and curling of a heated elliptical plate, *Proc. Roy. Soc.*, *A* 288, 396–417 (1965).
[14] E. H. Mansfield, Bending, buckling and curling of a heated thin plate, *Proc. Roy. Soc.*, *A* 268, 316–327 (1962).
[15] H-N. Chu and G. Herrmann, Influence of large Amplitudes on Free Flexural Vibrations of Rectangular Elastic Plates, *J. Appl. Mech.* 23, 532–540, (1956).